1. Flux density unit conversions.

In class, we discussed a quantity called the “flux” of a radiation field, \( F_\nu \), which has cgs units of \( \text{ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \). Sometimes \( F_\nu \) is referred to as “flux density” to distinguish it from the integrated quantity over frequency, which has units of \( \text{ergs cm}^{-2} \text{ s}^{-1} \).

An equivalent quantity is \( F_\lambda \), which has cgs units of \( \text{ergs cm}^{-2} \text{ s}^{-1} \text{ A}^{-1} \), and which is related to \( F_\nu \), by

\[
F_\nu \, d\nu = F_\lambda \, d\lambda.
\]

In high energy astrophysics, a commonly used equivalent quantity is \( F_E \), with units of photons \( \text{cm}^{-2} \text{ s}^{-1} \text{ keV}^{-1} \).

a. Derive the formulae, including evaluation of any numerical constants, which converts \( F_\lambda \) in cgs units to \( F_\nu \) in cgs units, to flux in Janskies, and to \( F_E \) in photons \( \text{cm}^{-2} \text{ s}^{-1} \text{ keV}^{-1} \).

b. Suppose a source of radiation has a spectral shape which is a “power law” in frequency, that is

\[
F_\nu \propto \nu^{-\alpha}
\]

where the constant \( \alpha \) is called the spectral index. Derive a formula which relates \( F_E \) in photons \( \text{cm}^{-2} \text{ s}^{-1} \text{ keV}^{-1} \), to the total integrated flux, \( F \), from 0.2 keV to 10 keV in units of \( \text{ergs cm}^{-2} \text{ s}^{-1} \).

2. Limb Darkening.

Consider a semi-infinite plane-parallel slab of gas. Determine the emergent intensity of radiation as a function of direction from the normal, \( I(\theta) \), where \( \theta \) is the angle between \( I \) and the normal. Assume the source function is \( S(\tau) = a + b\tau \), where \( \tau \) is the optical depth into the gas normal to the surface.

3. Opacity.

Calculate how far you could see through Earth’s atmosphere if it had the opacity of the solar photosphere. Assume the Sun’s opacity is \( \kappa = 0.264 \text{ cm}^2 \text{ g}^{-1} \), and the density of the Earth’s atmosphere is \( 1.2 \times 10^{-3} \text{ g cm}^{-3} \).

4. The Saha Equation.

Use the Saha equation to determine the fraction of hydrogen atoms that are ionized, \( N_{II}/N_{total} \), at the center of the Sun, where \( N_{total} = N_I + N_{II} \). Assume the temperature is 15.8 million degrees Kelvin, and the number density of electrons is \( n_e = 6.4 \times 10^{25} \text{ cm}^{-3} \). Approximate the ratio of the partition functions, \( Z_{II}/Z_I = 1 \). Does your result agree with the fact that practically all of the Sun’s hydrogen is ionized at the Sun’s center? Explain.