ASTRONOMY 300B
Homework No. 3
Due: in class, Wednesday, February 4, 2009

1. Flux density unit conversions.

In class, we discussed a quantity called the "flux" of a radiation field, F_ν, which has cgs units of ergs cm^{-2} s^{-1} Hz^{-1}. Sometimes F_ν is referred to as "flux density" to distinguish it from the integrated quantity over frequency, which has units of ergs cm^{-2} s^{-1}.

An equivalent quantity is F_λ, which has cgs units of ergs cm^{-2} s^{-1} $\AA^{-1}$, and which is related to F_ν, by

$$F_\nu \, d\nu = F_\lambda \, d\lambda.$$

In high energy astrophysics, a commonly used equivalent quantity is F_E, with units of photons cm^{-2} s^{-1} keV^{-1}.

a. Derive the formulae, including evaluation of any numerical constants, which converts F_λ in cgs units to F_ν in cgs units, to flux in Janskies, and to F_E in photons cm^{-2} s^{-1} keV^{-1}.

b. Suppose a source of radiation has a spectral shape which is a "power law" in frequency, that is

$$F_\nu \propto \nu^{-\alpha}$$

where the constant α is called the spectral index. Derive a formula which relates F_E in photons cm^{-2} s^{-1} keV^{-1}, to the total integrated flux, F, from 0.2 keV to 10 keV in units of ergs cm^{-2} s^{-1}.

2. Limb Darkening.

Consider a semi-infinite plane-parallel slab of gas. Determine the emergent intensity of radiation as a function of direction from the normal, $I(\theta)$, where θ is the angle between I and the normal. Assume the source function is $S(\tau)=a+b\tau$, where τ is the optical depth into the gas *normal* to the surface.

3. Opacity.

Calculate how far you could see through Earth's atmosphere if it had the opacity of the solar photosphere. Assume the Sun's opacity is $\kappa = 0.264$ cm^2 g^{-1}, and the density of the Earth's atmosphere is 1.2×10^{-3} g cm^{-3}.

4. The Saha Equation.

Use the Saha equation to determine the fraction of hydrogen atoms that are ionized, N_{II}/N_{total}, at the center of the Sun, where $N_{total} = N_I + N_{II}$. Assume the temperature is 15.8 million degrees Kelvin, and the number density of electrons is $n_e=6.4\times10^{25}$ cm^{-3}. Approximate the ratio of the partition functions, $Z_{II}/Z_I=1$. Does your result agree with the fact that practically all of the Sun's hydrogen is ionized at the Sun's center? Explain.