Astronomy 300B
Prof. Bechtold
Homework No. 7: Special Relativity
DUE: Monday, March 23, 2009 if you want the hw graded before the exam

1. The Twin Paradox. (10 points)

The following is based on a discussion of the twin paradox originally given by Sir Charles Darwin.

On New Year's Day, 1984, an astronaut (A) sets out from Earth at speed v=0.8c and travels to the nearest star, alpha-Centauri, which is (for the sake of argument) exactly 4 light-years away as measured in the Earth frame of reference. Having reached the star, he immediately turns around and returns to Earth at the same speed, arriving home on New Year's Day, 1994, by Earth time. The astronaut has a brother, B, who remains on Earth, and they agree to send one another greetings by radar-telephone on every New Year's Day until the traveler, A, gets back.

a. Show that A sends 6 messages (including the one made on the last day of his trip) whereas B sends 10.

b. Draw a space-time diagram of A's journey as plotted in the Earth's reference frame, i.e. plot x versus ct, and label both axes in light years.

c. Draw the world lines of all the radar signals that B transmits on the plot you made for part b. Verify with the help of the diagram that A has received only 1 signal up to the moment of his turnaround, and receives another 9 signals during the return half of the trip.

d. Draw another space-time diagram, again in the Earth reference frame, showing the world lines of the astronaut and of all the signals that *he* sends. Verify that his brother receives one message each 3 years of Earth time, for the first 9 years after his brother's departure and then receives 3 more messages during the last year, making a total of 6 -- which is just right, since for the astronaut the trip has taken 3 years each way.

2. SS433: The Transverse Doppler Effect. (10 points).

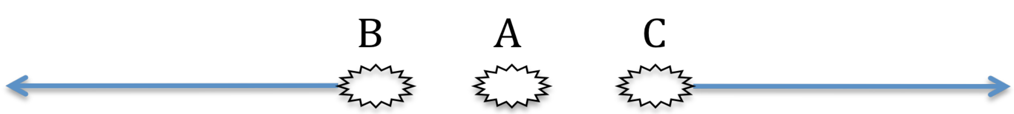
SS433 is an amazing X-ray binary system, which was discovered to have three sets of emission lines, one highly blueshifted, one highly redshifted, and one with almost no Doppler shift. Around the time of its discovery, SS433 was even satirized on Saturday Night Live, by Father Guido Sarducci who called it "the a-comin' and a-goin' star".

The description of the model which was implied by the observations is shown in Carroll and Ostlie, p. 735-737. On the class web site, I've scanned these pages in, as well as figures 17.26 and 17.27, which are necessary to do this problem. Please read this material.

The emission lines arise in a *precessing* jet generated by a neutron star in the binary.

Answer the following, which is 17.27 in Carroll & Ostlie:
The relativistic (v/c = 0.26) jets coming from the accretion disk in SS 433 sweep out cones in space as the disk precesses. The central axis of these cones makes an angle of 79 degrees with the line of sight, and the half-angle of each cone is 20 degrees. This means that at some point in the precession cycle, the jets are moving perpendicular to the line of sight. Yet, from Fig. 17.26, the radial velocities obtained from the Doppler-shifted spectral lines do *not* cross at zero radial velocity, but at v~10,000 km/sec. Use the equation we derived in class for the relativisitic Doppler effect to explain this discrepancy. You may ignore the speed of the SS 433 binary itself, which is only about 70 km/sec.

3. Addition of Velocities and the Doppler Effect. (10 points) Consider three galaxies, A, B, and C. An observer in A measures the velocities of C and B and finds they are moving in opposite directions, each with a speed of 0.8c relative to him. Thus, according to measurements in his frame, the distance between them is increasing at the rate 1.6c.



a. What is the speed of A observed in B? What is the speed of C observed in B?

b. In each galaxy, an observer has a radio transmitter which emits light at frequency ν in its own rest frame. What is the frequency of B's signals as received by A? What is the frequency of A's signals as received by C? What is the frequency of B's signals as received by C?

4. Transformation of Inertial Reference Frames (10 points)

Consider three inertial frames of reference K, K', and K''. Let K' move with velocity v with respect to K, and let K'' move with velocity v' with respect to K'. All velocities are co-linear.

a. Write the Lorentz transformation equations relating x,y,z,t with x',y',z',t' and also those relating x',y',z',t' with x'',y'',z'',t''. Combine these equations to get the relations between x,y,z,t and x'',y'',z'',t''.

b. Show that these relations are equivalent to a direct transformation from K to K'' in which the relative velocity v'' of K'' with respect to K is given by the relativistic addition relation:

5. Snapshots of a Cube and Sphere. (20 points).

Consider a cube, which in its rest frame has edges of length 1 meter. It is traveling with relativistic velocity v, in a direction parallel to one of its edges, with one face facing towards us, perpendicular to our line of sight. In our frame, the cube is Lorentz contracted in the direction of motion. This Lorentz contraction could be measured by placing 8 clocks at the vertices of the cube, and synchronizing them with 8 clocks in our frame. Then if we measure the dimensions of the cube *at the same time* with all clocks, we would measure a Lorentz contraction in the direction of motion.

a. A different question is what we would "see" if we took a snapshot of the cube. This is because a snapshot is a record of the light passing through our detector at a particular instant of time, in our frame. The light from different corners of the cube which enters our detector at a given time t, left the corners of the cube at different times in the past, since the light travel time to the farther corners is longer than the light travel time to the nearer corners. Show that in our snapshot, the cube appears *rotated*, and derive the angle of rotation in terms of velocity v.

b. Show that in a snapshot of a sphere, it appears to be a sphere, not an ellipsoid squished in the direction of motion.